Polarimetry Basics

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Outline

- **Polarimetry Definitions**
- **Polarimetry Principle**
- **Polarization Accuracy**
- **Polarimetry Advantages**
- **Astronomical Polarimetry**
- **Simple Polarimeters**
- **Polarimetric Measurements from IGO**

Polarimetry

- Polarimetry provides a totally different way of studying of electromagnetic radiations from celestial objects as compared to photometry or spectrometry
- Electromagnetic Radiation is described as:
  $\mathrm{E_x = E_{x0} e^{i(wt+kz+\delta x)}}$
  and
  $\mathrm{E_y = E_{y0} e^{i(wt+kz+\delta y)}}$
- Photometry & Spectroscopy provide information on:
  $\mathrm{I(intensity) = E_{x0}^2 + E_{y0}^2}$ as a function of wavelength λ

Polarimetry ... contd.

- Polarimetry provides information on:
  $\mathrm{I(intensity)} = \mathrm{E}_{\mathrm{x0}}^2 + \mathrm{E}_{\mathrm{y0}}^2$
  $\mathrm{Q} = \mathrm{E}_{\mathrm{x0}}^2 - \mathrm{E}_{\mathrm{y0}}^2$
  $\mathrm{U} = 2\mathrm{E}_{\mathrm{x0}}\mathrm{E}_{\mathrm{y0}}\cos(\delta\mathrm{x} - \delta\mathrm{y})$
  $\mathrm{V} = 2\mathrm{E}_{\mathrm{x0}}\mathrm{E}_{\mathrm{y0}}\sin(\delta\mathrm{x} - \delta\mathrm{y})$
  where I,Q,U.V are stokes parameters
- In principle one can allow light to fall on the detector through four different orientations of the axis of a dichroic filter and measure intensities I1, I2, I3 and I4 on the detector - this in turn gives estimates for I, Q, U, V

Muller Matrix

- The polarizing property of any light transmitting device can be described by any $\mathrm{M} = [4 \times 4]$ matrix
- S = I, Q, U, V stokes matrix of incoming beam
- $\mathrm{S}' = \mathrm{I}', \mathrm{Q}', \mathrm{U}', \mathrm{V}'$ stokes matrix of outgoing beam
- $\mathrm{ColumnMatrix}[\mathrm{S}'] = [\mathrm{M}] \times \mathrm{ColumnMatrix}[\mathrm{S}]$

Principles of Polarimetry

- Elements of Muller Matrix $= \mathrm{a_{ij}}(\alpha)$ where $\alpha =$ angle between axis of polarising device and electric vector of incident light
- $\mathrm{I}'(\alpha) = \mathrm{I.a_{11}}(\alpha) + \mathrm{Q.a_{12}}(\alpha) + \mathrm{U.a_{13}}(\alpha) + \mathrm{V.a_{14}}(\alpha)$
- Aligning the polarising device (viz. polaroid sheet, $\lambda/2$ plate, Wollaston prism etc.) in four different positions and measuring corresponding I' by a detector, one can estimate all the four I, Q, U, V
- Degree of Linear Polarization $\mathrm{p} = \frac{(\mathrm{Q}^2+\mathrm{U}^2)^{\frac{1}{2}}}{\mathrm{I}}$
- Position angle of Polarization Vector $\theta = \frac{1}{2}\tan^{-1}(\frac{\mathrm{U}}{\mathrm{Q}})$

Noise & Photometric Accuracy

- Noise can arise from (i) Instrument's detector thermal noise (ii) Atmospheric density fluctuations (iii) Intrinsic to the source – photon noise
- If S is signal above background B and N the noise then we require S/N > 1 for the signal to be detected above the noise
- S/N $\sim$ 3 is necessary to detect the signal with confidence - so called 3σ detection
- S/N depends of the signal integration time t and for random noise in a signal of n counts, the noise contributes to a mean error $\sigma_{\mathrm{n}} \sim \mathrm{n}^{\frac{1}{2}}$

Photometric Accuracy ... contd.

- S=Source photon counts B=Background photon counts M=S+B=Measured signal through and aperture
- B is measured from the same aperture on a nearby blank sky
- In practice, observations are made for time t, spending a fraction f 'on-source' and (1-f) on background i.e. 'off-source'
- Accumulated errors on M and B are $\sigma_{\mathrm{M}} \sim \mathrm{M}^{\frac{1}{2}}$ and $\sigma_{\mathrm{B}} \sim \mathrm{B}^{\frac{1}{2}}$
- Integration over longer t improves $\mathrm{S/N} \propto \sqrt{\mathrm{t}}$

Photometric Accuracy ... contd.

- In time ft we accumulate Mft counts with an error of $(\mathrm{Mft})^{\frac{1}{2}}$
- Estimate of $\mathrm{M} = \frac{1}{\mathrm{ft}}(\mathrm{Mft} \pm (\mathrm{Mft})^{\frac{1}{2}}) = \mathrm{M} \pm (\frac{\mathrm{M}}{\mathrm{ft}})^{\frac{1}{2}}$
- Thus uncertainty in M is reduced to $\sqrt{\mathrm{t}}$ thus to reduce the error by $1/2$, we need to integrate four times longer
- Thus in observing time t we have $\sigma_{\mathrm{M,t}} = \sqrt{(\mathrm{M}/\mathrm{ft})}$ and $\sigma_{\mathrm{B,t}} = \sqrt{(\mathrm{B}/(1-\mathrm{f})\mathrm{t})}$
- S=M-B thus error in S is $\sigma_{\mathrm{S}}^2 = \sigma_{\mathrm{M}}^2 + \sigma_{\mathrm{B}}^2$

Photometric Accuracy ... contd.

- After time t we have $\sigma^2_{\mathrm{S,t}} = \sigma^2_{\mathrm{M,t}} + \sigma^2_{\mathrm{B,t}} = \frac{\mathrm{M}}{\mathrm{ft}} + \frac{\mathrm{B}}{(1-\mathrm{f})\mathrm{t}}$
- Minimize error in S for a given time t by choosing f such that $\delta\sigma^2/\delta\mathrm{f} = 0$
- Thus $\frac{\delta\sigma^2_{\mathrm{S,t}}}{\delta\mathrm{f}} = -\frac{\mathrm{M}}{\mathrm{f}^2\mathrm{t}} + \frac{\mathrm{B}}{(1-\mathrm{f})^2\mathrm{t}}$
- σ^2 is minimized if $\frac{1-\mathrm{f}}{\mathrm{f}} = (\frac{\mathrm{B}}{\mathrm{M}})^{\frac{1}{2}}$
- *Very strong signal* B<<M and f$\sim$1 one can spend most of the time measuring M($\sim$S) since background correction is small and need not be accurately measured
- *Very weak signal* M$\sim$B and f$\sim \frac{1}{2}$ one can spend *equal* time observing background as well as the source since S=M-B$\sim$0 and this subtraction needs to be very accurate

Photometric Accuracy ... contd.

- Signal-to-Noise ratio is
$(\frac{\mathrm{S}}{\mathrm{N}})^2 = (\frac{\mathrm{S}}{\sigma_{\mathrm{St}}})^2 = \frac{(\mathrm{M}-\mathrm{B})^2}{\mathrm{M/ft}+\mathrm{B}/(1-\mathrm{f})\mathrm{t}} = \frac{(\mathrm{M}-\mathrm{B})^2\mathrm{ft}}{\mathrm{M}+\mathrm{fB}/(1-\mathrm{f})}$
- For an optimum f the total integration time required to attain a *given* S/N is the main goal since we need S/N>3 for a confident source detection
- Solving for t using the optimum f relation from above we get $\mathrm{t} = \frac{\mathrm{M}(1+(\mathrm{B/M})^{1/2})^2}{(\mathrm{M}-\mathrm{B})^2}(\frac{\mathrm{S}}{\mathrm{N}})^2$
- For a given source the time taken to reach a given S/N increases with the *square* of S/N thus continue integrating photons till S/N∼5 and not beyond

Photometric Accuracy ... contd.

- **Very strong signal** B<<M thus time taken to reach a given S/N is $\propto$ 1/M
- **Very weak signal** S(=M-B)<<M and M$\sim$B$\sim$constant thus $\mathrm{t} \propto \mathrm{B}/\mathrm{S}^2$ for a given S/N - thus it is very time-consuming to search for weak signal amongst noise - especially in presence of a bright background

Advantages of Polarimetry

- The values of Stokes parameters depend on relative values of intensities and thus absolute photometry need **NOT** be performed to determine I1, I2, I3, I4
- Airmass corrections, calibration by standard stars etc. are **NOT** required
- The uncertainty in p values are: $\delta \mathrm{p} \sim (\frac{\delta \mathrm{I}}{\mathrm{I}})$
- Suitable for objects which are bright and (or) have high polraization values (viz. comet, reflection nebulae etc.)

Polarimetry as a powerful diagnostic tool

- Various Astrophysical processes that produce **linear polarization**
- Dust scattering (single/multiple) produces linear/circular polarization viz. solar system : comets; nebulae and star forming clouds; circumstellar shell etc.
- Magnetic field (WD, neutron stars) produces polarization by Zeeman splitting
- Synchrotron emission $\Rightarrow$ polarization as in AGN.
- Electron scattering $\Rightarrow$ polarization as in the atmosphere of Be stars

Polarimetry as a powerful diagnostic tool ...contd.

- Various Astrophysical processes that produce **circular polarization**
- Multiple scattering - as in comet coma, stars embedded in dust cloud ($v = 10^{-4}$ may be expected)
- Due to Zeeman splitting by magnetic field - as in WD
- Twist in the grain alignment is possible - as in IS medium, by the change in the direction of aligning force (magnetic field)

Astronomical Polarimetry

- Polarimetry mainly consists of two classes
- Non-modualting polarimetry with low precision (1%)
- Modulator polarimetry with high precision (0.01%)
- In modulator polarimeter intensities of two orthogonal polarization beams are measured with the same detector, within a very short time interval

Simple Polarimeters

- Low precision polarimeter - simple dichroic sheet polarimeter light (from telescope) $\Rightarrow$ rotating polaroid sheet $\Rightarrow$ detector (CCD) $\mathrm{I}' = (\frac{\mathrm{I}}{2})(1 + \mathrm{pcos}(2\alpha))$
- High precesion polarimeter - light (from telescope) $\Rightarrow$ rotating $\lambda/2$ plate $\Rightarrow$ Wollaston prism $\Rightarrow$ detector (CCD) $\mathrm{I}' = \mathrm{I} \pm \mathrm{Qcos}(4\alpha) \pm \mathrm{Usin}(4\mathrm{sin}\alpha)$

An example of low precision polarimetry - Cometary Polarimetry

- A dichroic sheet polarizer may be used in front of the tube of the telescope (or in front of the detector)
- The detector can be a CCD, photographic plate for extended objects or SSP, PMT for point objects
- With the axis of polarizer at three different orientations (viz w.r.t. celestial NS) one can record intensity values (I1, I2, I3) on the detector

Comet Halley Observations in 1986 apparition

- Celestron-14" telescope was used at Mt.Abu with kodak photographic film as detector and simple dichroic sheet in front of the telescope tube (Sen et al. Icarus 1990, 86, p248)
- $\mathrm{p} = 2\frac{\sqrt{\mathrm{I1(I1-I2)+I2(I2-I3)+I3(I3-I1)}}}{\mathrm{(I1+I2+I3)}}$
- $\tan(2\theta) = \sqrt{3}\frac{\mathrm{(I3-I2))}}{\mathrm{(2I1-I2-I3)}}$
- Linear Polarization $\mathrm{p} = \frac{(\mathrm{Q}^2+\mathrm{U}^2)^{\frac{1}{2}}}{\mathrm{I}}$
- Position Angle $\theta = (\frac{1}{2})\tan^{-1}(\frac{\mathrm{U}}{\mathrm{Q}})$

Comet Halley Image in 1986

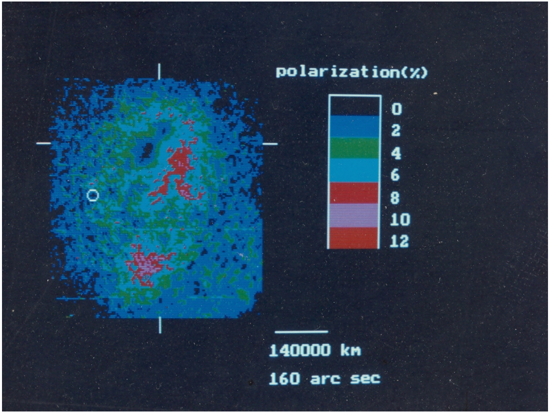


Figure 1: Comet Halley Image

Polarimetry of Comet Halley

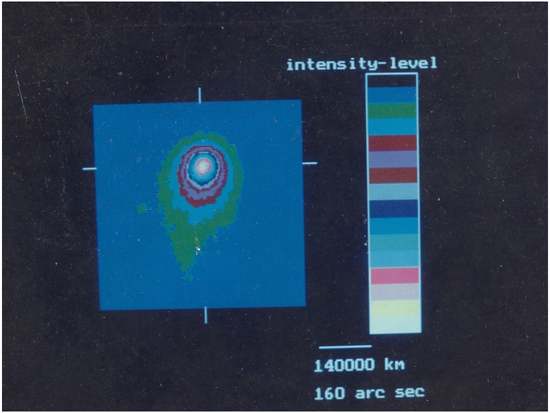


Figure 2: Polarimetry of Comet Halley

Polarimetry of Comet Halley ...contd.

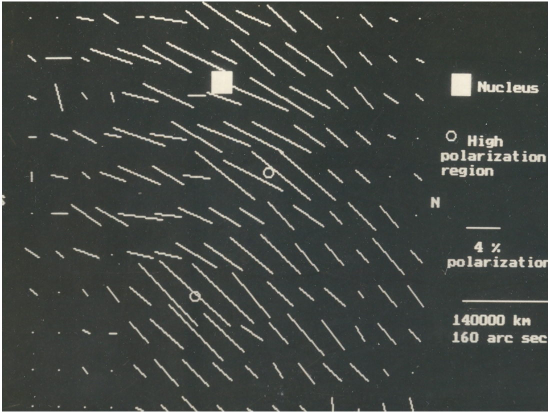


Figure 3: Polarimetry of Comet Halley at different epoch

Polarimetry of Comet 67P/C-G

Comet 67P/ C-G :

Our analysis indicated the presence of large particles just before and after the perihelion. Also there was post perihelion ejection of small fluffy aggregate grains. Results also indicated different grains being ejected at different hemishperes of nucleus. There was large variations of polarization in inner coma (< 3000 km), indicative of several ejections, which can be time resolved if better spatial resolutions are available. Whether grains have surface or sub-surface origin ?

(at IUCAA 2 m telescope, 1 pixel was corresponding to 370 km, <-> 400 pixels).

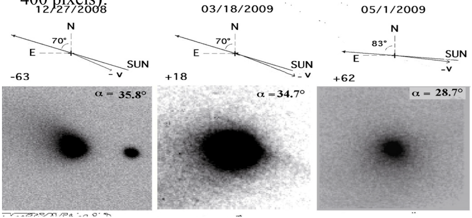


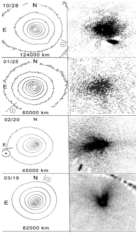
Figure 4: Polarimetry of Comet 67P/C-G

Comet C/2009

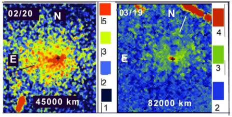
Comet C/2009 Garradd *results*

Comet Garradd was observed from IUCAA and OHP during Oct 2011 and March 2012, phase 28° to 35° (r-1.3 – 1.9), under Indo-French collaboration. Discovered in Aug 2009 / dynamically new comet (peri 23 Dec 2011, r=1.55) / Hale Bopp type –.
Jet activity is observed on each period.

2 or 3 months after perihelion, jets with higher polarization were observed



Evolution of jets from Oct 2011 to March 2012
Isophotes and enhanced intensity images



Polarization maps with higher polarization in February and March (butterfly structure in March)

1

Figure 5: Comet Garradd results

Dark Star Forming Cloud CB39

The distribution of polarization for stars background to star forming cloud CB39(Sen et al.2000, AnAS, v141, p175). Image taken with IUCAA polarimeter

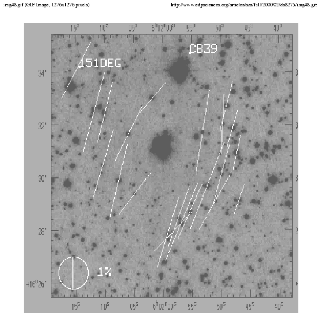


Figure 6: Polarimetry of CB39

Polarization Measurements from IGO polarimeter

Object	p(%)	Error in p(%)	θ°
c27CB-I	15.080	0.933	-22.510
C27CB-II	8.656	1.333	-22.112
c27CR	3.747	0.654	-23.745
c27R	1.302	0.444	-22.498
HD155	4.426	0.070	4.489
kof28	1.139	0.091	-22.492
arn69V	1.628	0.030	-22.493
kof29	24.475	1.424	31.343
C-G	48.854	0.224	-10.678
HD084	4.168	0.023	-41.829
kof30	5.617	0.027	17.501
kof01RI	9.723	0.403	-35.334
kof01R2	7.893	0.330	-14.783
kof01CR	9.334	2.161	-25.992

Table 1: Polarization Measurements carried out by IUCAA Polarimeter at IGO

Spectropolarimetry from IGO Polarimeter

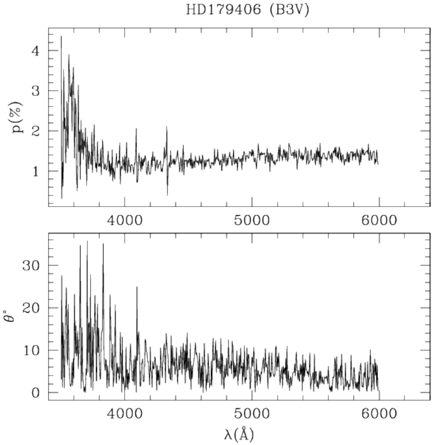


Figure 7: Spectropolarimetry of star HD179406

Imaging Polarimeter Design

A TYPICAL HIGH RESOLUTION POLARIMETR :
Design taken from Sen and Tandon (1994), Proceedins of SPIE bi-annual meeting, Hawai , USA

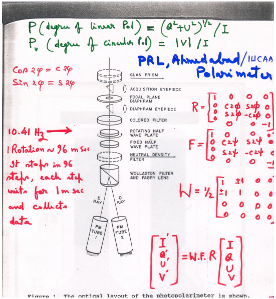


Figure 8: Imaging Polarimeter Design

Polarimeter Data reduction

- The HWP in the polarimeter is rotated in four discrete positions w.r.t to the North Galactic Pole i.e. $0^\circ, 22.5^\circ, 45^\circ \& 67.50^\circ$
- This yeilds $\mathrm{I}^{'}(0^\circ) = \frac{1}{2}(\mathrm{I} + \mathrm{Q})$; $\mathrm{I}^{'}(45^\circ) = \frac{1}{2}(\mathrm{I} - \mathrm{Q})$; $\mathrm{I}^{'}(22.5^\circ) = \frac{1}{2}(\mathrm{I} + \mathrm{U})$ & $\mathrm{I}^{'}(67.5^\circ) = \frac{1}{2}(\mathrm{I} - \mathrm{U})$
- Solving for I, Q and U we get: $\mathrm{Q} = \mathrm{I}^{'}(0^\circ) - \mathrm{I}^{'}(45^\circ)$; $\mathrm{U} = \mathrm{I}^{'}(22.5^\circ) - \mathrm{I}^{'}(67.5^\circ)$; $\mathrm{I} = \mathrm{I}^{'}(0^\circ) + \mathrm{I}^{'}(45^\circ)$ & $\mathrm{I} = \mathrm{I}^{'}(22.5^\circ) + \mathrm{I}^{'}(67.5^\circ)$
  Note: Measurement of I is redundant

Polarimeter Data reduction ...contd.

- Imaging Polarimeter provides 'e' and 'o' images (extraordinary and ordinary) for each star (each star image is split into e and o images due to the Wollaston Prism in the optical path) in the field and since there are images obtained at four discrete HWP rotation angles $0°, 22.5°, 45°\&67.50°$ – we get eight **magnitude** measurments for each star denoted as go0,ge0,go22,ge22,go45,ge45,go67 & ge67
- These need to be converted into **counts** i.e. $o0 = 10^{(-0.4go0)}$; $o22 = 10^{(-0.4go22)}$ etc.

Polarimeter Data reduction ...contd.

- Consider relations $\mathrm{r1} = \sqrt{\frac{\mathrm{e0/o0}}{\mathrm{e45/o45}}}$ & $\mathrm{r2} = \sqrt{\frac{\mathrm{e22/o22}}{\mathrm{e67/o67}}}$
- $\mathrm{q} = \frac{|\mathrm{r1}-1|}{\mathrm{r1}+1}$ & $\mathrm{u} = \frac{|\mathrm{r2}-1|}{\mathrm{r2}+1}$
- $\mathrm{p} = 100\sqrt{\mathrm{q}^2 + \mathrm{u}^2}$ & $\theta = \frac{57.3}{2}\tan^{-1}(\frac{\mathrm{u}}{\mathrm{q}})$

Interstellar Polarization

- To illustrate how star light is polarized by dust in the Interstellar Medium (ISM), consider an ensemble of elongated grains such as long cylinders
- Suppose that each grain is orientated with its long axis perpendicular to the direction of propagation of incident radiation: we may define $\mathrm{Q}_{\|}$ and $\mathrm{Q}_{\perp}$ as the values of the extinction efficiency $\mathrm{Q}_{\mathrm{ext}}$ when the E-vector is parallel and perpendicular to the long axis of the grain, respectively

Interstellar Polarization...contd.

- The anisotropy in physical shape introduces a corresponding anisotropy in extinction: because the E-vector sees an apparently larger grain in the parallel direction, we have $Q_{\parallel} > Q_{\perp}$
- The quantity $\Delta Q = Q_{\parallel} > Q_{\perp}$ is a measure of the resulting polarization. Note that polarization is, in general, small compared with extinction i.e. $Q_{\parallel} \sim Q_{\perp} >> \Delta Q$
- The amplitude (or degree) of linear polarization P is usually expressed as a percentage $P = 100 \left\{ \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right\}$ where I is the intensity

Interstellar Polarization...contd.

- When partially plane-polarized light from a star is observed, intensity maxima and minima i.e. $\mathrm{I_{max}}$ and $\mathrm{I_{min}}$ are recorded in orthogonal directions as the analyser is rotated
- These measurements allow the amplitude and position angle of the polarization vector to be determined
- An alternative definition is the polarization in magnitude units, denoted by the lower-case symbol $\mathrm{p} = 2.5\log\left\{\frac{\mathrm{I_{max}}}{\mathrm{I_{min}}}\right\}$

Interstellar Polarization...contd.

- Polarization may be described more generally in terms of the Stokes parameters I , Q, U and V. For partially plane-polarized light, these are given by:
- $\mathrm{I} = \mathrm{I}_{\mathrm{max}} + \mathrm{I}_{\mathrm{min}}$
- $\mathrm{Q} = \mathrm{PIcos2}(\theta_{\mathrm{G}} - 90)$
- $\mathrm{Q} = \mathrm{PIsin2}(\theta_{\mathrm{G}} - 90)$
- $\mathrm{V} = 0$

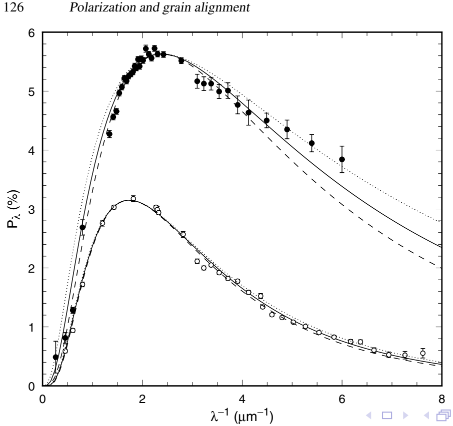
Interstellar Polarization...contd.

- where θ_{G} is defined as the angle between the E-vector and the direction of the North Galactic Pole, measured counterclockwise (toward increasing galactic longitude) on the sky
- The linear component is described by Q and U, which can also be expressed in magnitude units
- $\mathrm{q} = \mathrm{pcos2}(\theta_{\mathrm{G}} - 90)$
- $\mathrm{u} = \mathrm{psin2}(\theta_{\mathrm{G}} - 90)$

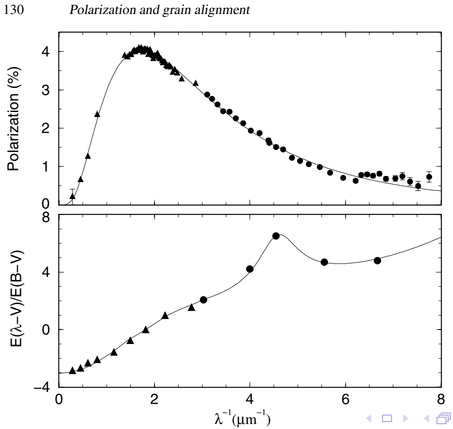
The Serkowski law

- The spectral dependence of linear polarization or polarization curve (usually plotted as P_λ versus λ^{-1}) displays a broad, asymmetric peak in the visible region for most stars
- The wavelength of maximum polarization, $\lambda_{\max}$, varies from star to star and is typically in the range $0.3\mu\mathrm{m}$ to $0.8\mu\mathrm{m}$ with a mean value of $0.55\mu\mathrm{m}$
- The dependence of P_λ on λ is well described by the Serkowski emperical law (formula) as:
  $\mathrm{P}_\lambda = \mathrm{P}_{\max}\mathrm{exp}\left\{-\mathrm{K}\mathrm{ln}^2\left\{\frac{\lambda_{\max}}{\lambda}\right\}\right\}$
- The parameter K, which determines the width of the peak in the curve, was initially taken to be constant with a value of K=1.15

Interstellar Polarization Optical for HD204827 and HD99872 with peaks at $\lambda_{\max} = 0.42\mu\mathrm{m}$ and $0.58\mu\mathrm{m}$ respectively

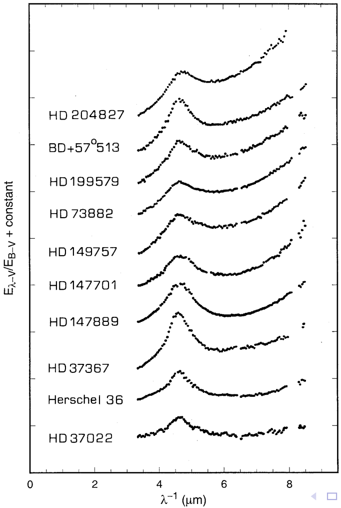


UV Polarization (model fit by Serkowski formula: K=1.09, $P_{max} = 4.03\%$ and $\lambda_{max} = 0.59\mu m$) and Extinction for line of sight to HD161056

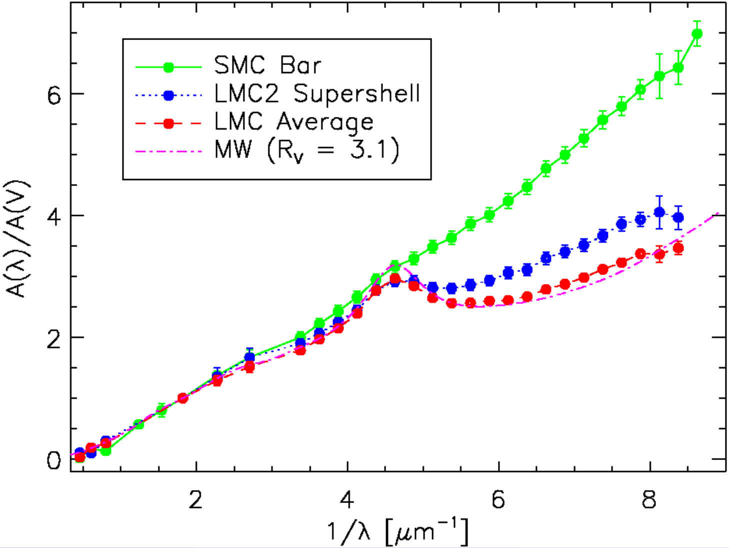


IUE UV Extincion curves in various directions of the galaxy

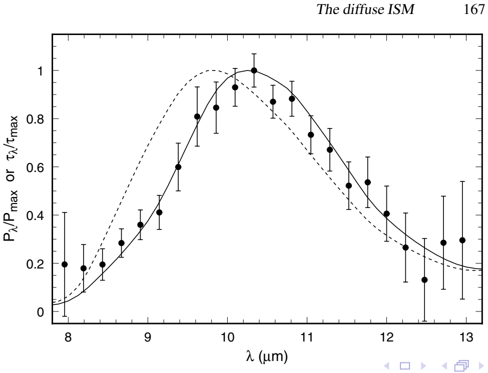
86 *Extinction and scattering*



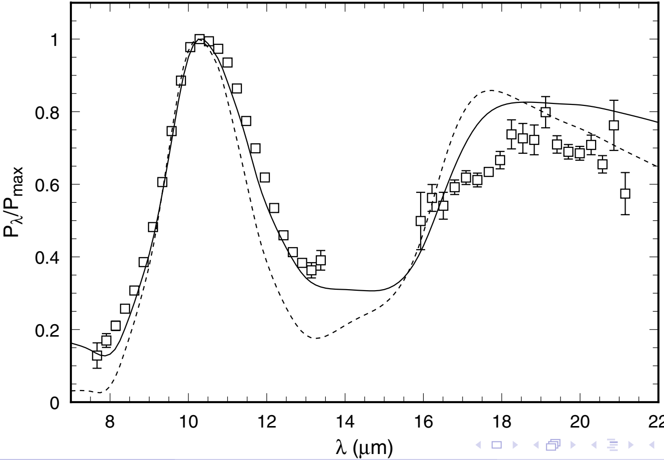
Interstellar Extinction Curves for MW, LMC and SMC



NIR Polarization (average observations of two early stars WR48A and GL2104) and model with oblate spheroids of axial ration 2.1 and amorphous olivine grains of size a=$0.1\mu m$

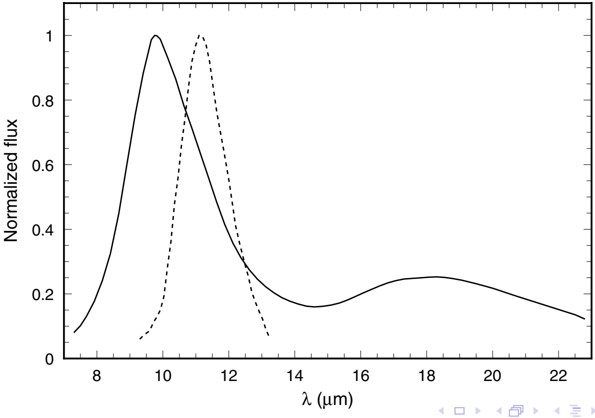


NIR Polarization Observations of BN object and model fits



NIR Silicate Emission of IRAS M-type star (full curve) and SiC emission on Carbon star (broken curve)

240 *Dust in stellar ejecta*



Interstellar Extinction

- The degree of reddening or selective extinction of a star is quantified as $\mathrm{E_{B-V} = (B-V) - (B-V)_0}$ in the Johnson photometric system, where (B-V) and $\mathrm{(B-V)_0}$ are observed and intrinsic values of the colour index and $\mathbf{E_{B-V}}$ is the **colour excess**
- As the extinction is always greater in the B filter (central wavelength $0.44\mu\mathrm{m}$) than in V ($0.55\mu\mathrm{m}$, $\mathrm{E_{B-V}}$ is a positive quantity for reddened stars and zero (to within observational error) for unreddened stars
- Intrinsic colours are determined as a function of spectral type by studying nearby stars and stars at high galactic latitudes that have little or no reddening

Interstellar Extinction... contd.

- The relationship between total extinction at a given wavelength and a corresponding colour excess depends on the wavelength-dependence of extinction, or extinction curve
- In the Johnson system, the extinction in the visual passband may be related to $\mathrm{E_{B-V}}$ by $\mathrm{A_V = R_V E_{B-V}}$ where $\mathrm{R_V}$ is termed the ratio of total to selective visual extinction
- Theoretically, $\mathrm{R_V}$ is expected to depend on the composition and size distribution of the grains

Interstellar Extinction... contd.

- However, in the low-density ISM, $\mathrm{R_V}$ has been shown to be virtually constant and a value of $\mathrm{R_V} \sim 3.05 \pm 0.15$ may be assumed for most lines of sight
- To observe the extinction curve,the most reliable and widely used technique involves the pairing of stars of identical spectral type and luminosity class but unequal reddening and determining their colour difference The apparent magnitude of each star as a function of wavelength may be written as:
$\mathrm{m_1 = M_1(\lambda) + 5logd_1 + A_1(\lambda)}$
$\mathrm{m_2 = M_2(\lambda) + 5logd_2 + A_2(\lambda)}$

Interstellar Extinction... contd.

- Where M, d and A represent absolute magnitude, distance and total extinction, respectively and subscripts 1 and 2 denote reddened and comparison stars
- The intrinsic spectral energy distribution, represented by $\mathrm{M}(\lambda)$, is expected to be closely similar or identical for stars of the same spectral classification, thus we may assume $\mathrm{M}_1(\lambda) = \mathrm{M}_2(\lambda)$
- If $\mathrm{A}(\lambda) = \mathrm{A}_1(\lambda) >> \mathrm{A}_2(\lambda)$ i.e. the extinction towards the star 2 is negligible compared with that towards the star 1, then the magniture difference $\Delta\mathrm{m}(\lambda) = \mathrm{m}_1(\lambda) - \mathrm{m}_2(\lambda)$ reduces to

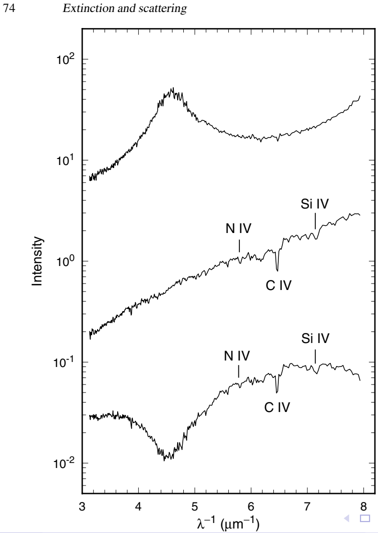
Interstellar Extinction... contd.

- $\Delta\mathrm{m}(\lambda) = 5\log\left\{\frac{\mathrm{d}_1}{\mathrm{d}_2}\right\} + \mathrm{A}(\lambda)$
- The first term on the right-hand side of this equation is independent of wavelength and constant for a given pair of stars
- Hence, the quantity $\Delta\mathrm{m}(\lambda)$ may be used to represent $\mathrm{A}(\lambda)$
- The constant may be eliminated by means of normalization with respect to two standard wavelengths λ_1 and λ_2:
- $\mathrm{E}_{\mathrm{norm}} = \frac{\Delta\mathrm{m}(\lambda)-\Delta\mathrm{m}(\lambda_2)}{\Delta\mathrm{m}(\lambda_1)-\Delta\mathrm{m}(\lambda_2)} = \frac{\mathrm{A}(\lambda)-\mathrm{A}(\lambda_2)}{\mathrm{A}(\lambda_1)-\mathrm{A}(\lambda_2)} = \frac{\mathrm{E}(\lambda-\lambda_2)}{\mathrm{E}(\lambda_1-\lambda_2)}$

Interstellar Extinction... contd.

- where $\mathrm{E}(\lambda - \lambda_2)$, is the difference in extinction between the specified wavelengths, which is equal to the colour excess
- The normalized extinction $\mathrm{E}_{\mathrm{norm}}$ should be independent of stellar parameters and determined purely by the extinction properties of the interstellar medium
- Extinction curves are commonly normalized with respect to the B and V passbands in the Johnson system, i.e. the normalized extinction equation becomes $\frac{\mathrm{E}_{\lambda-\mathrm{V}}}{\mathrm{E}_{\mathrm{B}-\mathrm{V}}}$

Pair method illustration with two stars (IUE spectra)



Pair method illustration with two stars (IUE spectra)

An illustration of the pair method for determining interstellar extinction curves. The lower curve is the ultraviolet spectrum of a reddened star (HD34078, spectral type O9.5 V, $\mathrm{E_{B-V}} = 0.54$) and the middle curve is the corresponding spectrum of an almost unreddened star of the same spectral type (HD38666, $\mathrm{E_{B-V}} = 0.03$). A few representative spectral lines are labelled. The vertical axis plots intensity in arbitrary units note that the scale is logarithmic and hence equivalent to magnitude. The upper curve is the resulting extinction curve, obtained by taking the intensity ratio (equivalent to magnitude difference) of the two spectra

Pair method ...contd.

- The relative extinction (replacing the labels λ_1 and λ_2 with B and V) is:
- $\frac{\mathrm{E}(\lambda - \lambda_2)}{\mathrm{E}(\lambda_1 - \lambda_2)} = \frac{\mathrm{E}_{\lambda-\mathrm{V}}}{\mathrm{E}_{\mathrm{B}-\mathrm{V}}} = \frac{\mathrm{A}_\lambda - \mathrm{A}_\mathrm{V}}{\mathrm{E}_{\mathrm{B}-\mathrm{V}}} = \mathrm{R}_\mathrm{V} \left\{ \frac{\mathrm{A}_\lambda}{\mathrm{A}_\mathrm{V}} - 1 \right\}$
- Thus the **absolute** extinction $\mathrm{A}_\lambda/\mathrm{A}_\mathrm{V}$ may be deduced from the relative extinction if $\mathrm{R}_\mathrm{V} = \mathrm{A}_\mathrm{V}/\mathrm{E}_{\mathrm{B}-\mathrm{V}}$, the **ratio of total-to-selective extinction** is known

Thanks